

# New Bayesian analysis of hybrid EoS constraints with mass-radius data for compact stars

A. Ayriyan<sup>a1</sup>, D.E. Alvarez-Castillo<sup>b,c</sup>, D. Blaschke<sup>b,d</sup>, H. Grigorian<sup>a,e</sup>,  
M. Sokolowski<sup>d</sup>

<sup>a</sup> *Laboratory of Information Technologies, JINR Dubna, Russia*

<sup>b</sup> *Bogoliubov Laboratory of Theoretical Physics, JINR Dubna, Russia*

<sup>c</sup> *Instituto de Física, Universidad Autónoma de San Luis Potosí, México*

<sup>d</sup> *Institute of Theoretical Physics, University of Wroclaw, Poland*

<sup>e</sup> *Department of Physics, Yerevan State University, Armenia*

## Abstract

We suggest a new Bayesian analysis using disjunct mass and radius constraints for extracting probability measures for cold, dense nuclear matter equations of state. One of the key issues of such an analysis is the question of a deconfinement transition in compact stars and whether it proceeds as a crossover or rather as a first order transition. The latter question is relevant for the possible existence of a critical endpoint in the QCD phase diagram under scrutiny in present and upcoming heavy-ion collision experiments.

## 1 Introduction

The most basic features of a neutron star (NS) are the radius  $R$  and the mass  $M$  which so far have not been well determined simultaneously for a single object. In some cases masses are precisely measured like in the case of binary systems, but radii are quite uncertain [1]. In the other hand, for isolated neutron stars some radius and mass measurements exist but lack the necessary precision to allow conclusions about their interiors. In fact, it has been conjectured that there exists a unique relation between  $M$  and  $R$  for all neutron stars and their equation of state (EoS), thus constraining the properties of their interior [2]. For this reason, accurate observations of masses and radii are crucial to study cold dense nuclear matter expected to exist in neutron stars.

However, the presently observational data allow to make only a probabilistic estimations of the internal structure of the star, via Bayesian analysis (BA). This technique has been applied for the first time to this problem by

---

<sup>1</sup>Email: ayriyan@jinr.ru

Steiner et al. [3] who exemplified very well the power of this method. In their analysis, however, only a particular type of objects (X-ray bursters) has been considered under strongly model dependent assumptions. In this work is a continuation of our preliminary probabilistic studies of the super-dense stellar matter equation of state using Bayesian Analysis and modeling of relativistic configurations of neutron stars [4, 5] We put special emphasis on the choice of observational constraints and focus on investigations of the possible existence of deconfined quark matter in massive neutron stars such as the recently observed  $2 M_{\odot}$  pulsars [6, 7].

## 2 NS structure and EoS

The microscopical properties of compact stars are modeled in the framework of general relativity, where the Einstein equations are solved for a static (non-rotating), spherical star resulting in the Tolman–Oppenheimer–Volkoff (TOV) equations [8, 9, 10]

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r) \quad (1)$$

$$\frac{dp(r)}{dr} = -G \frac{(\varepsilon(r) + p(r))(m(r) + 4\pi p(r)r^3)}{r(r - 2Gm(r))} \quad (2)$$

as well as the equation for the baryon number profile

$$\frac{dn_B(r)}{dr} = 4\pi r^2 m_N \frac{n_B(r)}{\sqrt{1 - 2Gm(r)/r}} \quad (3)$$

These equations are integrated from the center of the star towards its surface, with the radius of the star  $R$  being defined by the condition  $p(R) = 0$  and the gravitational mass by  $M = m(R)$ . In a similar manner, the baryon mass of the star is given by  $M_B = m_N n_B(R)$ , where  $m_N$  is the nucleon mass.

To complete the solution to the TOV equations the EoS is required. It is given by the relation  $p = p(\varepsilon)$  which carries information about the microscopic ingredients of the dense nuclear matter, as mentioned before. Thus, the above equations have to be solved simultaneously using the equation of state under boundary conditions at the star centre ( $r = 0$ ), taken as an input. In this way, for a given value of  $\varepsilon(r = 0)$  the solution of the TOV equations are the  $p(r)$  and  $m(r)$  profiles and with them the parametric relationship  $M(R)$  can be obtained.

For the present study we use the scheme suggested by Alford, Han and Prakash [11] for defining the hybrid EoS (shorthand: AHP scheme),

$$p(\varepsilon) = p_h(\varepsilon)\Theta(\varepsilon_H - \varepsilon) + p_h(\varepsilon_H)\Theta(\varepsilon - \varepsilon_H)\Theta(\varepsilon_H + \Delta\varepsilon - \varepsilon) + p_q(\varepsilon)\Theta(\varepsilon - \varepsilon_H - \Delta\varepsilon), \quad (4)$$

where  $p_h(\varepsilon)$  is a hadronic matter EoS and  $p_q(\varepsilon)$  represents the high density matter phase, here considered as deconfined quark matter with the bag model type EoS

$$p_q(\varepsilon) = c_q^2 \varepsilon - B. \quad (5)$$

The stiffness of this EoS is given by  $c_q^2$ , the squared speed of sound. The positive bag constant  $B$  assures confinement, i.e. the dominance of the hadronic EoS at low densities. Note that a parametrization in this form by Haensel et al. [12] describes pretty well the superconducting NJL model derived in [13] and systematically explored in [14]. In the AHP scheme, the hadronic EoS is fixed and not subject to parametric variations. However, we shall use different well-known model EoS in our study the nonrelativistic variational EoS of Akmal et al. [15] (APR) and the density-dependent relativistic meanfield EoS of Typel and Wolter [16] (DD2) in its parametrization from Ref. [17]. Both EoS come eventually with extensions due to an excluded volume for baryons, as described for DD2 in [18].

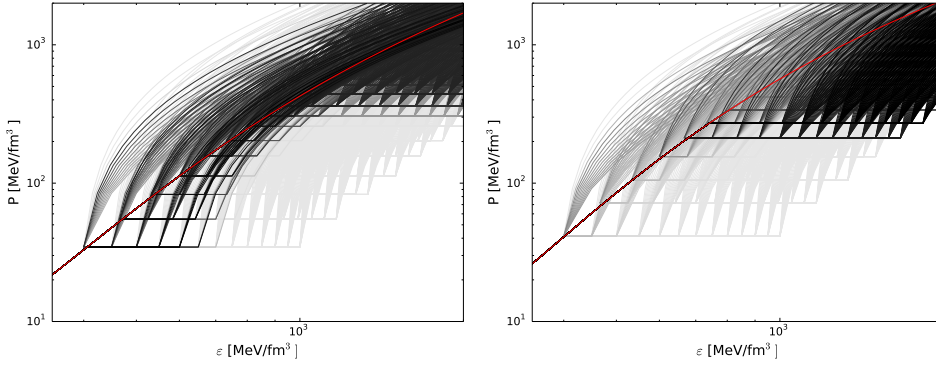


Figure 1: Varying parameters of the hybrid EoS in the AHP scheme for the case of APR as hadronic EoS without (left) and with (right) excluded volume for baryons.

The free parameters of the model are the transition density  $\varepsilon_H$ , the energy density jump  $\Delta\varepsilon \equiv \gamma\varepsilon_H$  and  $c_q^2$ . The set of hybrid EoS in the plane pressure versus energy density is shown in Fig. 1 for APR as the hadronic EoS.

### 3 BA Formulation and Formalization

We define the vector of free parameters  $\vec{\pi}(\varepsilon_c, \gamma, c_q^2)$  defining the hybrid EoS with a first order phase transition from nuclear to quark matter.

These parameters are sampled

$$\pi_i = \vec{\pi}(\varepsilon_k, \gamma_l, c_{qm}^2), \quad (6)$$

where  $i = 0 \dots N - 1$  (here  $N = N_1 \times N_2 \times N_3$ ) as  $i = N_1 \times N_2 \times k + N_2 \times l + m$  and  $k = 0 \dots N_1 - 1$ ,  $l = 0 \dots N_2 - 1$ ,  $m = 0 \dots N_3 - 1$ , here  $N_1$ ,  $N_2$  and  $N_3$  denote the number of parameters for  $\varepsilon_k$ ,  $\gamma_l$  and  $c_{qm}^2$ , respectively. Solving the TOV equations with varying boundary conditions for  $\varepsilon(r = 0)$  generates a sequence of  $M(R)$  curves characteristic for each of these  $N$  EoS. Subsequently, different neutron star observations with their error margins can be used to assign a probability to each choice in from the set of EoS parameters. We use three constraints: (i) the mass constraint for PSR J0348+0432 [7], (ii) the radius constraint for PSR J0437-4715 [19] and (iii) the constraint on the baryon mass at the well measured gravitational mass for the star B in the double pulsar system PSR J0737-3039 [20], which improved the earlier suggestion by Podsiadlowski et al. [21].

The goal is to find the set of most probable  $\pi_i$  based on given constraints using Bayesian Analysis (BA). For initializing BA we propose that *a priori* each vector of parameter  $\pi_i$  has probability equal one:  $P(\pi_i) = 1$  for  $\forall i$ .

#### 3.1 Mass constraint for PSR J0348+0432

We propose that the error of this measurement is normal distributed  $\mathcal{N}(\mu_A, \sigma_A^2)$ , where  $\mu_A = 2.01 M_\odot$  and  $\sigma_A = 0.04 M_\odot$  are measured for PSR J0348+0432 [7]. Using this assumption we can calculate conditional probability of the event  $E_A$  that the mass of the neutron star corresponds to this measurement

$$P(E_A | \pi_i) = \Phi(M_i, \mu_A, \sigma_A), \quad (7)$$

where  $M_i$  is the maximum mass obtained for  $\pi_i$  and  $\Phi(x, \mu, \sigma)$  is the cumulative distribution function for the normal distribution.

#### 3.2 Radius constraint for PSR J0437-4715

Recently, a radius constraint for the nearest millisecond pulsar PSR J0437-4715 have been obtained [19] giving  $\mu_B = 15.5$  km and  $\sigma_B = 1.5$  km. With

these data one calculates the conditional probability of the event  $E_B$  that the radius of a neutron star corresponds to this measurement

$$P(E_B | \pi_i) = \Phi(R_i, \mu_B, \sigma_B). \quad (8)$$

### 3.3 $M$ – $M_B$ Relation Constraint for PSR J0737-3039(B)

This constraint gives a region in the  $M$ – $M_B$  plane. We need to estimate the probability of a point  $\mathcal{M}_i = (M_i, M_{B_i})$  to be close to the point  $\mu = (\mu_G, \mu_B)$ . The mean values  $\mu_G = 1.249$ ,  $\mu_B = 1.36$  and standard deviations  $\sigma_M = 0.001$ ,  $\sigma_{M_B} = 0.002$  are given in [20]. The probability can be calculated by following formula:

$$P(E_K | \pi_i) = [\Phi(\xi_G) - \Phi(-\xi_G)] \cdot [\Phi(\xi_B) - \Phi(-\xi_B)], \quad (9)$$

where  $\Phi(x) = \Phi(x, 0, 1)$ ,  $\xi_G = \frac{\sigma_M}{d_M}$  and  $\xi_B = \frac{\sigma_{M_B}}{d_{M_B}}$ ,  $d_M$  and  $d_{M_B}$  are absolute values of components of vector  $\mathbf{d} = \mu - \mathbf{M}_i$ , here  $\mu = (\mu_G, \mu_B)^T$  was given in [20] and  $\mathbf{M}_i = (M_i, M_{B_i})^T$  stems for the solution of TOV equations for the  $i^{\text{th}}$  vector of EoS parameters  $\pi_i$ . Note that formula (9) does not correspond to a multivariate normal distribution.

### 3.4 Calculation of *a posteriori* Probabilities

Note, that these measurements are independent on each other. Therefore, the complete conditional probability of the event that a compact object constructed with an EoS characterized by  $\pi_i$  fulfils all constraints is

$$P(E | \pi_i) = P(E_A | \pi_i) \times P(E_B | \pi_i) \times P(E_K | \pi_i). \quad (10)$$

Now, we can calculate probability of  $\pi_i$  using Bayes' theorem:

$$P(\pi_i | E) = \frac{P(E | \pi_i) P(\pi_i)}{\sum_{j=0}^{N-1} P(E | \pi_j) P(\pi_j)}. \quad (11)$$

## 4 Results and Discussion

We apply the scheme of BA for probabilistic estimation of the EoS given by vector parameter  $\vec{\pi}$ . Varying the three parameters in the intervals  $400 < \varepsilon[\text{MeV}/\text{fm}^3] < 1200$ ,  $0 < \gamma < 1.5$  and  $0.3 < c_s^2 < 1.0$  with  $N_1 = N_2 = N_3 =$

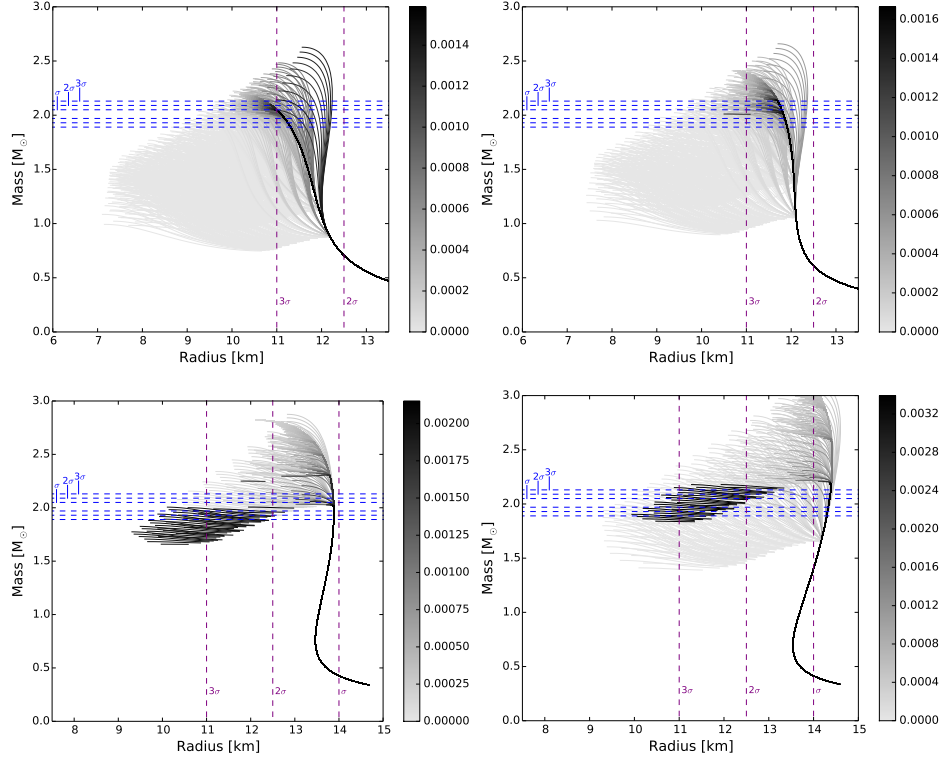


Figure 2: **Top row:** Mass-radius sequences for the AHP scheme with the APR EoS for baryonic matter. Only for the hybrid EoS with baryonic excluded volume effect high mass twin star configurations are obtained (right panel). **Bottom row:** Same as top row with the DD2 EoS and both cases accounting for a baryonic excluded volume, the larger one in the right panel. High-mass twin configurations are a rather typical feature for these stiff hadronic EoS combined with sufficiently stiff quark matter ones. The greyscale of the lines indicated the probability for the corresponding EoS.

13 we explore a set of  $N = 2197$  hybrid EoS for each choice of hadronic EoS. The results for the mass-radius sequences are presented in Fig. 2 where the greyscale of the lines indicates the probability for the corresponding EoS parameter set under the above constraints. The four panels of Fig. 2 differ only in the choice of hadronic EoS. The top row shows results for the APR-based hybrid EoS without (left) and with (right) baryonic excluded volume effect. Only for the latter case high mass twin stars are obtained which lie on the sequence of the so-called "third family" of compact stars, separated from

the second one (ordinary neutron stars) by a set of unstable configurations not shown in the figure. Hybrid EoS based on the stiffer DD2 EoS are used to obtain the sequences shown in the bottom row, with the larger excluded volume in the right panel. As can be seen in Fig. 2, for the stiffer DD2-based hybrid EoS it is quite typical to achieve a third family of compact star sequences at the high mass of  $2M_{\odot}$ . Actually, in the BA suggested here, these twin star configurations have the highest probability measure. In order to support these high-mass twins, the excluded volume correction should be introduced (see, e.g., [22, 23, 18]).

Note that the occurrence of high-mass twins is testable by observations. It requires precise radius measurements of  $2M_{\odot}$  pulsars in order to verify the existence of an almost horizontal branch of high-mass pulsars with almost the same mass but radii differing by up to a few kilometers. As a necessary condition for such sequences the hadronic star sequence should have radii exceeding 12 km. The observation of this striking high-mass twin phenomenon would indicate a strong first-order phase transition in neutron star matter which gives important constraints for searches of a critical endpoint in the QCD phase diagram.

## 5 Acknowledgment

This research was partially supported by Bogoliubov-Infeld Program and by the Ter-Antonian-Smorodinsky Program for bilateral collaboration of JINR Dubna with Universities and Institutes in Poland and in Armenia, resp.; and by the COST Action MP1304 "NewCompStar". A.A. acknowledges JINR grant No. 14-602-01. D.A. and D.B. were supported by the Polish Narodowe Centrum Nauki (NCN) under grant No. UMO-2011/02/A/ST2/00306.

## References

- [1] M. C. Miller. *Astrophysical Constraints on Dense Matter in Neutron Stars*. [arXiv:1312.0029 [astro-ph.HE]].
- [2] L. Lindblom. *Limits on the Gravitational Redshift from Neutron Stars*. The Astrophysical Journal (1984), vol. 278, 364-368.
- [3] A. W. Steiner, J. M. Lattimer and E. F. Brown. *The Equation of State from Observed Masses and Radii of Neutron Stars*. The Astrophysical Journal (2010), vol. 722, 33.

- [4] D. Alvarez-Castillo, A. Ayriyan, D. Blaschke and H. Grigorian. *Bayesian Analysis of Hybrid EoS based on Astrophysical Observational Data*. LIT Scientific Report 2011-2013, JINR Publishing Department, Dubna (2014) pp. 123-126. Editors: Gh. Adam, V.V. Korenkov, D.V. Podgany, T.A. Strizh, P.V. Zrelov. ISBN 978-5-9530-0381-0.
- [5] D. B. Blaschke, H. A. Grigorian, D. E. Alvarez-Castillo and A. S. Ayriyan. *Mass and radius constraints for compact stars and the QCD phase diagram*. J. Phys. Conf. Ser. (2014) vol. 496, p. 012002. [arXiv:1402.0478 [astro-ph.HE]].
- [6] P. Demorest, T. Pennucci, S. Ransom, M. Roberts and J. Hessels, *Shapiro Delay Measurement of A Two Solar Mass Neutron Star*. Nature (2010), vol. 467, p. 1081.
- [7] J. Antoniadis, P. C. C. Freire, N. Wex, T. M. Tauris, R. S. Lynch, M. H. van Kerkwijk, M. Kramer, C. Bassa, V. S. Dhillon, T. Driebe, J. W. T. Hessels, V. M. Kaspi, V. I. Kondratiev, N. Langer, T. R. Marsh, M. A. McLaughlin, T. T. Pennucci, S. M. Ransom, I. H. Stairs, J. van Leeuwen, J. P. W. Verbiest, D. G. Whelan. *A Massive Pulsar in a Compact Relativistic Binary*. Science (2013), vol. 340, p. 6131.
- [8] R. C. Tolman. *Static Solutions of Einstein's Field Equations for Spheres of Fluid*. Physical Review (1939), vol. 55, p. 364.
- [9] J. R. Oppenheimer and G. M. Volkoff, *On Massive neutron cores*. Physical Review (1939), vol. 55, p. 374.
- [10] N. K. Glendenning. *Compact stars: nuclear physics, particle physics, and general relativity*. New York: Springer, 2000.
- [11] M. G. Alford, S. Han and M. Prakash. *Generic conditions for stable hybrid stars*. Physical Review D (2013), vol. 88, no. 8, p. 083013.
- [12] J.L. Zdunik and P. Haensel. *Maximum mass of neutron stars and strange neutron-star cores*. Astronomy and Astrophysics (2013), vol. 551, p. A61.
- [13] T. Klöhn, D. Blaschke, F. Sandin, C. Fuchs, A. Faessler, H. Grigorian, G. Röpke and J. Trümper, *Modern compact star observations and the quark matter equation of state*. Phys. Lett. B **654**, 170 (2007).



- [14] T. Klähn, R. Lastowiecki and D. B. Blaschke. *Implications of the measurement of pulsars with two solar masses for quark matter in compact stars and heavy-ion collisions: A Nambu–Jona-Lasinio model case study*. Physical Review D (2013), vol. 88, no. 8, 085001.
- [15] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall. *Equation of state of nucleon matter and neutron star structure*. Physical Review C (1998), vol. 58, no. 3, pp. 1803-1838.
- [16] S. Typel and H. H. Wolter. *Relativistic mean field calculations with density dependent meson nucleon coupling*. Nuclear Physics A (1999), vol. 656, 331.
- [17] S. Typel, G. Röpke, T. Klähn, D. Blaschke and H. H. Wolter. *Composition and thermodynamics of nuclear matter with light clusters*. Physical Review C (2010), vol. 81, p. 015803.
- [18] S. Benic, D. Blaschke, D. E. Alvarez-Castillo, T. Fischer and S. Typel. *A new quark-hadron hybrid equation of state for astrophysics - I. High-mass twin compact stars*. Astronomy and Astrophysics (2014), (submitted); [[arXiv:1411.2856 \[astro-ph.HE\]](#)].
- [19] S. Bogdanov. *The Nearest Millisecond Pulsar Revisited with XMM-Newton: Improved Mass-radius Constraints for PSR J0437-4715*. The Astrophysical Journal (2013), vol. 762, iss. 2, pp. 96.
- [20] F.S. Kitaura, H.-Th. Janka and W. Hillebrandt. *Explosions of O-Ne-Mg cores, the Crab supernova, and subluminal type II-P supernovae*. Astronomy and Astrophysics (2006), vol. 450, no. 1, pp. 345-350.
- [21] P. Podsiadlowski, J. D. M. Dewi, P. Lesaffre, J. C. Miller, W. G. Newton and J. R. Stone, *The Double pulsar J0737-3039: Testing the neutron star equation of state*. Monthly Notices of the Royal Astronomical Society (2005), vol. 361, p. 1243.
- [22] D. Blaschke, D.E. Alvarez-Castillo, S. Benic, G. Contrera, R. Lastowiecki. *Nonlocal PNJL models and heavy hybrid stars*. PoS ConfinementX (2012), 249; [[arXiv:1302.6275 \[hep-ph\]](#)].
- [23] D. Blaschke, D. E. Alvarez-Castillo and S. Benic, *Mass-radius constraints for compact stars and a critical endpoint*. PoS CPOD 2013 (2013), 063; [[arXiv:1310.3803 \[nucl-th\]](#)].